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Simple and Accurate Operators Based on Taylor Expansion for 2D Elastic Seismogram Calculation under Geological Discontinuities with Regular Cartesian Grids

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SUMMARY

We propose a set of local operators to deal with internal discontinuities that do not coincide with collocated Cartesian grids in 2D transversely isotropic media. We use globally optimally accurate operators in order to obtain high accuracy. We derive modified operators by extrapolating wavefields from nearby grid points to the discontinuous point, introducing boundary conditions; we then distribute those conditions to the nearby grid points. Numerical examples suggest that the operators improve the coherency of the wavefront. We would like to optimise the local operators to control all the error propagation during the modelling.

Introduction

Seismic waveform inversion is emerging as a powerful tool to reveal the Earth's interior and has been already applied to real data in order to obtain a fine structure, taking account for all the wave-equation effects including finite-frequency effects as well as high-order interactions with internal and outer interfaces. Since waveform inversion tries to fit the synthetics to observed data, it is crucial to be capable of controlling the precision in the forward modelling scheme. On the other hand, since we are obliged to perform a number of iterations of inversions to obtain a good convergence, due to the highly non-linear nature of the problem, we need an efficient forward modelling scheme that is easily adaptive to any model of the Earth's interior. A plausible seismic waveform inversion result thus follows an accurate and efficient forward modelling of seismic wavefield for a given Earth's model.

While family members of finite element methods such as spectral element methods (e.g. Komatitsch & Vilotte 1998) and Galerkin methods (e.g. Käser & Dumbser 2006) enjoy their increasing popularity especially in global seismology, despite their highly expensive computational efforts both for meshing and wavefield calculation itself since they can accurately handle any type of topography by carefully defining unstructured grid mesh, exploration seismology normally prefers finite difference methods (e.g. Madariaga 1976; Virieux 1984) since it is easy to implement and fast in calculation for any model without special costly meshing schemes but controlling the accuracy of finite difference operators, especially under the presence of lithological discontinuities and free surfaces that do not coincide with computational grids, has remained one of the well-known problems yet difficult to deal with. There have been some attempts to concur the difficulty (e.g. Moczo et al. 1993; Schönberg & Muir 1989; Lombard & Piraux 2004). However, some of them suffer from computational costs or shortcomings of accuracy.

In this study, we propose a set of operators to deal with internal discontinuities for collocated Cartesian grids. The main reasons of not using staggered grids (e.g. Virieux 1984) is that we have a great interest to optimise those operators by optimally minimising the mixed errors in partial derivatives in time and space in the near future, which will not be possible with staggered grids (see Geller & Takeuchi 1998). Other advantages of staying in collocated grids is that it is much more straightforward to treat discontinuities and anisotropy. Here in this paper, we focus on the local modified operators to eliminate numerical diffractions due to an ad-hoc stair-step approximation. This study is an extension of Mizutani (2002), who derived modified operators for internal interfaces in 2D SH case, to a 2D anisotropic elastic case.

Discretised version of equation of motion

First we introduce a strong form of equation of motion (for a general 3D case):

$$\rho \frac{\partial^2}{\partial t^2} u_i - \left(C_{ijkl} u_{k,l}\right)_{,j} = f_i \tag{1}$$

with u_i the displacement, ρ the density, C_{ijkl} the elastic moduli. $_{,i}$ denotes the spatial partial derivative. Einstein's summation is implied. The discretised version in collocated finite difference of the equation of motion is thus:

$$(A_{m_1 m_2 m_3 M n_1 n_2 n_3 N} - K_{m_1 m_2 m_3 M n N}) u_{n_1 n_2 n_3 N} = f_{m_1 m_2 m_3 M} \tag{2}$$

where we discretise u the displacement as:

$$u_{n_1 n_2 n_3 N} = u(n_1 \Delta x_1, n_2 \Delta x_2, n_3 \Delta x_3, N \Delta t) \tag{3}$$

A is an operator for $\rho \partial^2 u / \partial t^2$, **K** is an operator for $\left(C_{ijkl} u_{k,l}\right)_{,j}$.

Global Optimally accurate operators

Here we briefly review the concept of optimally accurate operators. There have been some studies on the stability of one-step time-domain discretised schemes (e.g, Geller & Takeuchi 1998; Geller et al. 2012). In order to maximise the accuracy with the limited band of finite difference operator, it is important to rearrange operators. We realise the rearranging the operators by evaluating the operator error for conventional operators that have normally no spatial dependency for time derivatives $\mathbf{A}^{\text{CONV}}$ and no time dependency for spatial derivatives $\mathbf{K}^{\text{CONV}}$. We seek to find the optimally accurate operators $\mathbf{A}^{\text{OPT}}$ and $\mathbf{K}^{\text{OPT}}$ that have both time and spatial dependencies. Implicit schemes must be approximately transformed into predictor-corrector schemes so that we do not need any modification for the main part of the computation, which is conventionally coded and thus very simple and efficient:

$$(\mathbf{A}^{\text{CONV}} - \mathbf{K}^{\text{CONV}})\mathbf{u}^0 = \mathbf{f} \tag{4}$$

$$(\mathbf{A}^{\text{CONV}} - \mathbf{K}^{\text{CONV}})\mathbf{u}^1 = (\mathbf{A}^{\text{OPT}} - \mathbf{A}^{\text{CONV}} - \mathbf{K}^{\text{OPT}} + \mathbf{K}^{\text{CONV}})\mathbf{u}^0 \tag{5}$$

where we sum up $\mathbf{u}^0$ and $\mathbf{u}^1$ to obtain more accurate wavefield. We have recently uploaded O(2,2) and (2,4) optimally accurate operators for a test use in Github (Cuvilliez & Fuji 2015). We use 2D global optimally accurate operators in this study but have not yet optimised the local operators for interfaces at this moment.

Locally modified operators for discontinuities

In order to seek explicit form of modified operators that accurately treat inter-node internal discontinuities, we extrapolate displacements from the grids to the interfaces, applying boundary conditions from left and right, then we translate them to the nearby grids. Modified operators for 2D transversely isotropic media can be derived in the same manner as 1D SH case (as shown in Mizutani 2002) except that we deal with at least nine point stencils and that due to mixed-derivatives terms appearing and thus we need more boundary conditions. Curvature of the inter-media interface within the grid cell has an effect of higher-order then is used in ongoing study. Hence, it will not contribute sufficiently and could be neglected. So we are allowed to replace an interface of arbitrary shape within a grid cell with its linear approximation.

At the lithological boundary in 2D anisotropic case following boundary conditions should be applied:

$$u_i|_-^+ = 0 \tag{6}$$

$$\bigtriangledown_{||} u_i|_-^+ = 0 \tag{7}$$

$$\bigtriangledown_{||} \bigtriangledown_{||} u_i|_-^+ = 0 \tag{8}$$

$$n_j C_{ijkl} u_{i,j}|_-^+ = 0 \tag{9}$$

$$\bigtriangledown_{||} \left(n_j C_{ijkl} u_{i,j} \right) |_-^+ = 0 \tag{10}$$

$$\left(\frac{C_{ijkl} u_{i,j}}{\rho} \right)_{,j} |_-^+ = 0 \tag{11}$$

where eq. 6 shows that the *displacement is continuous* on the boundary(subtract of displacements from both sides of discontinuity have to be equal to zero). *Traction is continuous* at the boundary (eq. 9). Continuity conditions for *first derivatives in direction parallel to the interface* for components of displacements (eq. 7) and stresses (eq. 10). Continuity of *second derivative of displacement* along the boundary (eq. 8). *Jump condition* on the boundary (eq. 11), derived form equation of motion using continuity of traction and displacement. Differential operator for the gradient in the direction parallel to the boundary: $\bigtriangledown_{||} = \bigtriangledown u - (n \cdot \bigtriangledown u)\, n$.

In order to realise the calculation for 2D media, we rewrite boundary conditions (eq-s. 6-11) could be written down in matrix form. Following expression represents boundary conditions and links left and right sides of the lithological interface:

$$B^- U_0 = B^+ U_1 \tag{12}$$

Then if to introduce Taylor series term we get full expression

$$B^{-}A^{-}U_0 = B^{+}A^{+}U_1 \tag{13}$$

Here, B^{-} and B^{+} are 12×12 matrices with boundary conditions for left and right side of the boundary respectively A^{-} and A^{+} are 12×12 matrices presenting Taylor expansion terms for extrapolation. U_0 and U_1 are 12 elements column-vectors that represent displacement values in two points containing lithological interface between them.

Finally, extraction of desired value U_0 from eq. 13 is straight forward:

$$U_0 = \underbrace{(B^{-}A^{-})^{-1}A^{+}B^{+}}_{C}\, U_1 \tag{14}$$

By explicitly finding C, we modify spatial derivative operators locally.

Numerical example

In this section we show numerical examples. We calculated wavefields with simple 2D media that consists of two anisotropic homogeneous regions separated with oblique lithological interface (Figs. 1 and 2). Propagation velocities in the left part are higher then in the right, with reflection and transmission coefficients over the boundary $R = 0.29$ $T = 0.71$. Absorbing boundary is applied and the source is a Ricker wavelet, placed in the slower part of the medium and marked with small light circle.

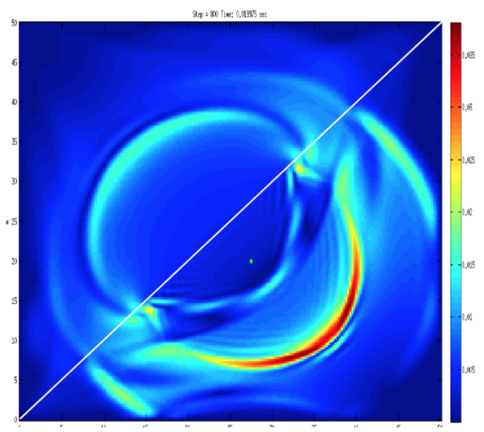

Figure 1 *Wavefield snapshot for conventional operators for an oblique discontinuity.*

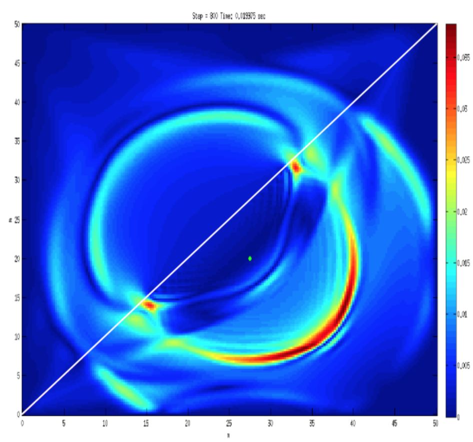

Figure 2 *Localised modified operator with globally optimally accurate operators*

Fig. 1 shows a snapshot of conventional operators and Fig. 2 for the operators globally optimally accurate and locally modified for interfaces. We can see that the conventional operators have some incoherent wavefronts especially in the slower medium due to the bad treatment of discontinuities. We observe some ringings on the faster medium whereas in the Fig. 2 the wavefront is much sharper and the reflected waves from the discontinuity is well coherent.

Conclusions and discussions

We have derived and developed a set of operators optimally accurate globally and modified for boundary conditions on internal discontinuities in a 2D media so that we can more accurately calculate synthetics even with finite difference regular grids. The fact that we have not yet optimised locally modified operators might cause some errors and this should be further examined quantitatively. The interest of this method is that we can improve the accuracy of existent finite difference schemes without making a lot of efforts. By being able to control the error propagation along some discontinuities with strong contrast, we will eliminate errors in synthetics without tremendous computational efforts and that will enable us to conduct waveform inversion more efficiently and accurately.

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